

PA – MDC GRADE 4 FRACTION BUCKETS

This lesson is intended to help you assess how well students compare two fractions with different numerators and different denominators by comparing them to benchmarks of 0, $\frac{1}{2}$ and 1.

This lesson is modeled from a unit taken from North Carolina Department of Public Instruction and revised by the PA-MDC Writing Committee.

*Concept
Development
Formative
Assessment
Lesson*

With Sincere thanks and appreciation for the effort and work of the Members of the PA- MDC Writing Committee and for the unwavering support from their home districts: Camp Hill School District, Cumberland Valley School District, Lower Dauphin School District and Shippensburg School District.

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FRACTION BUCKETS

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should then be used to inform the instruction that will take place for the remainder of your unit.

Mathematical goals: This lesson is intended to help you assess how well students compare two fractions with different numerators and different denominators by comparing them to benchmarks of 0, $\frac{1}{2}$ and 1. Students will also be able to explain their reasoning and strategies used in comparing fractions.

Common Core State Standards This lesson involves a range of mathematical practices from the standards, with emphasis on: Numbers and Operations in Fractions

- **4.NF.1** Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- **4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

This lesson involves a range of *Standards for Mathematical Practice*, with emphasis on:

MP.2. Reason abstractly and quantitatively.

MP.3. Construct viable arguments and critique the reasoning of others.

PA Core Standards:

PA CC2.1.4.C.1: Extend the understanding of fractions to show equivalence and ordering.

Introduction:

This lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work and create questions for students to answer in order to improve their solutions.

- Teacher will assign student pairs to work in small homogenous groups (partners/pairs) on a collaborative task to sort and assign fractions to an appropriate category. Throughout their work, students justify and explain their decisions to their partner.
- The class will share the process and outcomes with a whole class discussion.
- Teacher will return the pre-test and distribute post- test for students to improve upon their original answer.

Time Needed: estimated 90 – 120 minutes

Pre-assessment (15 minutes)

Whole Class Lesson and Collaborative Activity (40 – 45 minutes)

Whole Class Discussion (10 minutes)

Improving Solutions (Post-Test 15 minutes)

Timings given are only approximate. Exact timings will depend on the needs of the class. All students need not to finish all card sets to complete the lesson.

Materials required: It is recommended that all cards are laminated, cut and prepared ahead of time. It is recommended that the several sets of fraction cards are made of different colors or numbered on blank side to keep sets together.

- Each pair of students will need:
- Fraction Cards
- Fraction Bucket Cards

Graphic organizer is optional. It is useful if you will extend lesson to more than one class period or want to be able review answers formatively.

Before the Lesson

Pre-Assessment Task: *Fraction Sort* (15 minutes)

Framing the Task:

*Today we will work on a task to see how well you are able determine if a fraction fits under 1 of the following 5 categories: **Between One Half ($\frac{1}{2}$) and One Whole (1), Less than One Half ($< \frac{1}{2}$), More than One Whole (> 1), One Half ($= \frac{1}{2}$), or One Whole ($= 1$).***

Explain/Show your thinking on the lines provided. You will have 15 minutes to work independently on the task, “Fraction Sort.” After 15 minutes, I will collect your papers and see how you explained your decisions. Do not worry if you do not complete the task or if you have difficulty explaining your decisions. We will have a lesson which should help you in gaining more understanding.

Pre-Assessment Task: **Fraction Sort**

Name: _____

Determine if each fraction is: Between One Half and One Whole, Less than One Half, More than One Whole, One Half, or One Whole.

5/3	Explain your thinking:
2/2	Explain your thinking:
1/3	Explain your thinking:
4/8	Explain your thinking:
6/10	Explain your thinking:

Have students do this task individually in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have. You will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the pre-assessment. Students should use their understanding of fractions to determine the best fit and explain their thinking.

Students should not worry too much if they do not understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently.

Assessing Students' Responses

Collect students' responses to the task. Make some notes about what their work reveals about their current levels of understanding, and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some questions on the following page may serve as examples. These questions have been drawn from commonly identified student misconceptions. These can be written on the board (or projected) at the end of the lesson before the students revisit the initial task.

We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be helpful to the majority of students. These can be written on the board at the end of the lesson.

Below is a list of common issues and questions/prompts that may be written on individual initial tasks or during the collaborative activity to help students clarify and extend their thinking.

Common Issues:	Suggested Questions and Prompts:
Student does not understand greater than/less than.	<ul style="list-style-type: none"> • Which is larger (greater) \$0.50 or \$0.75? • What do the denominator and numerator of a fraction tell you? • How does the numerator influence your thinking as to the greater or less than value of two fractions with the same denominator being compared? ($a/b < > = c/b$) • How do you determine the greater than/ less than value of a fraction when compared to another fraction with a different denominator? ($a/b < > = c/d$)

Does not understand that a fraction represents a number larger than one whole.	<ul style="list-style-type: none"> • What does it mean if the numerator is larger than the denominator? • Can you draw a picture to represent $5/3$?
Does not understand the concept of equivalent fractions.	<ul style="list-style-type: none"> • Can you draw a number line and place $2/4$ and $6/12$ on that number line? What do you notice?

Suggested Lesson Outline

Whole Class Lesson

Improve individual solutions to the assessment task (15 – 20 minutes)

Return your students' work on the *Fraction Sort* problems. If you have not added questions to the students' work, write (or project) a short list of your most common questions to their misconceptions on board. Use these questions to guide a discussion prior to the students starting the collaborative activity with a partner.

- *What strategies did you use to determine the best fit for sorting the fractions into the correct category?*

Class will listen to two or three different approaches. Predetermine which students will share based on the pre-assessments. Pause to have students reflect by asking them to consider the following:

- *Was your approach to sorting the fractions similar to someone that shared? Does one method seem more efficient than others?*

If more clarification is needed draw students' attention to the questions you have written (or projected), to think about as they answer this question:

- *"If I had two fractions $2/5$ and $2/3$, how would I know which one is closer to 1? Or closer to $1/2$?"*
- *Can you prove your answer on a ...number line? Show me*
- *Can you draw a model to represent your answer? Show me*
- *Can you explain in words? Write and share*

Ask the class if they agree or disagree with these answers. Do you have any questions for your class mates? If not, move forward to the Collaborative Activity.

Collaborative Activity: Fraction Buckets

Homogeneously group students based on the results of the pre-assessment. Students will work in pairs.

Framing the Collaborative Activity:

- Each pair of students should receive a copy of the “Bucket Cards”, a set of “Fraction Cards and graphic organizer.
- Students lay out the bucket cards in the order of least to greatest. Shuffle the fraction cards and place them face down in front of themselves. Take turns flipping over a fraction card and place them under the correct bucket. Remind students to place fraction card in a row under the heading so each card is visible to the teacher as she rotates around the room.
- As a card is being placed under the bucket, the student must explain why they are choosing that particular bucket. If the partner agrees with the explanation record the fraction on the graphic organizer. The partner can then flip another card and the students continue. If the partner does not agree, they get a turn to explain where they think it goes. Both students must agree on which bucket card the fraction card will be placed. If the pair cannot agree, they can place the fraction card to the side for later discussion.
- Continue with card placement until allotted time expires. *Focus is not upon completion of all the cards but in the understanding of the task.*

As the students do this:

- Circulate around the room to observe the students work. Listen to students reasoning as they place a card. Ask a student to re-explain why a card was placed under a particular bucket. It should be about correct cards, as well as incorrect. Objective is to have students defend their thinking.
- If there is a card that is not agreed upon, listen to both arguments, and guide students (ask leading questions) to help them make a final decision. Try not to make suggestions that move students toward a particular approach. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.
- *Make a note of student approaches to the task.* You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. Partners should be engaged in checking their partner, asking for clarification, and taking turns. When calling on students make sure you allow the struggling groups to share first.

NOTE: If you notice several groups are having difficulty starting or are struggling on the same issue, then you (or another student pair) may want to demonstrate how to place a fraction card with a bucket card.

To plan for individual differences, as a pair of students struggle, remove the cards that are not $\frac{1}{2}$ or 1. Then, as they become more familiar with these cards, introduce the ‘Less than Half’ fraction cards, followed by ‘More than half’. Finally add the ‘More than One Whole’.

Extension activities

Extension 1: Give students blank fraction cards and have them create fractions for their partners to place.

Extension 2: On a separate sheet of paper (or math journals) have students write rules for each bucket.

Extension 3: Divide the cards between two students. Place the cards face down. Each student takes their first card and places it in the correct buckets. The student with the largest card takes their opponents card. If a card is misplaced it is automatically forfeited. If there is a tie, a second card is drawn, and the winner takes all the cards.

Whole-class discussion (10 minutes)

Summarize/conclude the lesson by discussing and generalizing what has been learned. Students sharing the strategies they used aloud will be valuable to the learning of the group. The generalization involves extending what has been learned to new examples.

- What have you learned today?
- Can you describe your method to us?
- Did you hear a group describe a more effective method than your strategy?

Improving individual solutions to the assessment task (10 minutes)

Return the initial task *Fraction Sort*, to students as well as the post-assessment task.

Look at your original responses and think about what you have learned during this lesson. Using what you have learned, try to improve your work.

If you have not added feedback questions to individual pieces of work then write (or project) your list of questions on the board.

Pre-Assessment Task: Fraction Sort

Name _____

Determine if each fraction is: Between One Half ($\frac{1}{2}$) and One Whole (1), Less than One Half ($< \frac{1}{2}$), More than One Whole (> 1), One Half ($= \frac{1}{2}$), or One Whole ($= 1$).

$$\frac{5}{3}$$

Explain/Show your thinking.

$$\frac{2}{2}$$

Explain/Show your thinking.

$$\frac{1}{3}$$

Explain/Show your thinking.

$$\frac{4}{8}$$

Explain/Show your thinking.

$$\frac{6}{10}$$

Explain/Show your thinking.

Fraction Sort

Name _____

Determine if each fraction is: Between One Half ($\frac{1}{2}$) and One Whole (1), Less than One Half ($< \frac{1}{2}$), More than One Whole (> 1), One Half ($= \frac{1}{2}$), or One Whole ($= 1$).

$$\frac{3}{3}$$

Explain/Show your thinking.

$$\frac{8}{5}$$

Explain/Show your thinking.

$$\frac{7}{14}$$

Explain/Show your thinking.

$$\frac{2}{5}$$

Explain/Show your thinking.

$$\frac{3}{5}$$

Explain/Show your thinking.

**Between One Half
and One Whole**



**Less than
One Half**

**More than
One Whole**



One Half

One Whole



Fraction Cards (page 1)

$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{1}{3}$
$\frac{2}{3}$	$\frac{3}{3}$	$\frac{6}{3}$	$\frac{1}{4}$
$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$
$\frac{1}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
$\frac{6}{5}$	$\frac{9}{5}$	$\frac{2}{6}$	$\frac{3}{6}$

Fraction Cards (page 2)

$\frac{5}{6}$	$\frac{6}{6}$	$\frac{2}{8}$	$\frac{4}{8}$
$\frac{5}{8}$	$\frac{8}{8}$	$\frac{11}{8}$	$\frac{3}{10}$
$\frac{5}{10}$	$\frac{9}{10}$	$\frac{10}{10}$	$\frac{3}{12}$
$\frac{6}{12}$	$\frac{9}{12}$	$\frac{12}{12}$	$\frac{10}{20}$

Graphic Organizer

Between One Half and One Whole

Less than One Half

More than One Whole

One Half

One Whole

Solutions

Between One Half and One Whole

	$\frac{2}{3}$	$\frac{3}{4}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$

Less than One Half

$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{2}{6}$	$\frac{2}{8}$	$\frac{3}{10}$
	$\frac{3}{12}$	

More than One Whole

$\frac{5}{2}$	$\frac{6}{3}$	$\frac{5}{4}$
$\frac{6}{5}$	$\frac{9}{5}$	$\frac{11}{8}$
	$\frac{3}{2}$	

One Half

$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$
$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$
	$\frac{10}{20}$	

One Whole

$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$
$\frac{6}{6}$	$\frac{8}{8}$	$\frac{10}{10}$
	$\frac{12}{12}$	

